A Review of an
Integrated High School Mathematics Program

At La Habra High School, we are now using an integrated sequence in Algebra 1 and piloting the second year of the sequence in some Geometry sections. Next year all the Geometry sections will use the second year of the integrated sequence and we will have two pilot sections of the third year of the sequence. This year I have taught the first year of the sequence.

The authors (Rubenstein, Craine and Butts (the texts are Integrated Math 1, 2 and 3, published by McDougall Littell) claim that over a three year period their trio of texts *teaches the same mathematical topics as a contemporary Algebra 1/Geometry/Algebra 2 sequence*. I set out to list the topics I missed from Algebra 1 and let math and science teachers know when students can be expected to have had some exposure to them. What I found leads me to believe the statement is not true, even allowing that what constitutes a "contemporary sequence" has always varied somewhat.

Absolute value equations and inequalities are never solved. The definition of limit in calculus depends on three absolute value inequalities, and the definitions of derivative and integral depend on limits. The point-slope formula which we so regularly use to find tangents and normals is buried in one problem in the second year of the sequence; it might not even be assigned. Completing the square, which we need for some integrals by trigonometric substitution, is not used to derive the quadratic formula (which is just announced as a rule as math texts did thirty years ago) but is buried without explanation in four problems. I could not find a single problem involving reduction of algebraic fractions. If a student wants to evaluate a simple limit such as $\lim \dfrac{x^2-49}{2x-14}$ as $x \to 7$ she'll have to wait for L'Hospital's Rule or estimate it on a calculator, which is not a problem-free process. Too bad.

Joint variation is not mentioned. So if a science teacher says work varies jointly as time and power or momentum varies jointly as mass and velocity, students may not understand what that means.

Students are asked to multiply binomials a bit before they factor trinomials in the process of graphing a parabola or solving a quadratic equation, but are only asked to square a couple of binomials in the midst of work on the binomial theorem in the second year of the sequence. Solving proportions generally is confined to those with monomial terms. About four scattered problems are as complex as $\frac{x+3}{5}=\frac{x-7}{3}$. There's no simplifying of such radical expressions as the following: $\frac{\sqrt{10}}{\sqrt{5}}$ $\frac{1}{\sqrt{3}}$ $7\sqrt{2}+5\sqrt{2}$. In the second year when the sin of 45 degrees is determined, it's $\frac{1}{\sqrt{2}}$ and never $\frac{\sqrt{2}}{2}$. Negative exponents come with scientific notation and a couple of "rules" in the first course. In the second and third courses negative and fractional exponents are applied to variables with only about two dozen problems of each type strung out over two years.

Classic applications of algebra to work, money, rate-time-distance and mixtures are gone. None of that has anything to do with the real world, of course. The authors' puffery, blithely accepted by fans of the sequence, claims that students can now "solve problems that are more realistic and more interesting". We're asked to calculate probabilities that a salvage boat will anchor directly over a sunken barge in a river and that a bead dropped into a grocery bag will fall through a circular hole cut out of the bottom. Those are so realistic: I've taken a salvage boat out to hunt for a barge in a river dozens of times this year alone, and I plan to stay up late tonight cutting 4" circular holes in the bottoms of my grocery bags. A colleague with more courage than I have set out to make statistical sampling more realistic by having the students mark a sample of lima beans, place them back in the larger population, and sampling again. The students did not compare ratios of marked beans to the sample size as instructed, but just kept marking all the beans.

Most of the people teaching the first year course skipped a chapter on

> *the Pythagorean Theorem, square roots, if-then statements and converses, geometric probability, surfaces of prisms, cylinders and pyramids, volumes of prisms, pyramids, cylinders and cones and areas and volumes of similar figures*

to get to a chapter on

> *reflections, parabolas translated from* $y=\pm x^2$*, exponents, multiplying binomials, exponents, factoring trinomials, parabolas* $y=(x+a)(x+b)$ *and the quadratic formula.*

Now they may find insufficient time to cover all of the skipped chapter. They are probably not aware that the quadratic formula is covered again in the second text of the sequence. The curricular problem is that the material on surface areas and volumes is only repeated in about six problems in later years of the sequence, although areas and volumes of spheres are covered. (I did not find surface area of a cone, but did not look long.) So those who make it to calculus some day may have trouble finding the rate of change of a volume or surface area of a cone, given the rates of change of its radius and height. Any student who changes from a customary to an integrated curriculum or vice versa will slip completely through a large crack and see some topics more than once and never see others. The number of students involved is not small and the problem is not trivial. Has it ever been seriously addressed by any decision makers?

I have not had the time to check inclusion of the material of a contemporary geometry or second year algebra course in this three year sequence. A former colleague I saw at a conference says conic sections are not included, and they are not in our text sequence. So we can't talk very meaningfully about light or radio waves passing through the focus of an automobile headlight or a satellite dish because the students do not know what a focus of a parabola is. On the other hand we looked at about a dozen scatter plots and decided they showed negative, no or positive correlation, even though they have no knowledge of a formula for computing correlation, let alone standard deviation.

It's my impression that a couple of my colleagues who had sets of TI 81's handy showed the kids how to push buttons to compute correlation. I'm a bit leery of mindless button pushing after seeing students in some low level classes compute a quotient like 75/15 on a calculator: about 40% of those who do will give an answer of .2 instead of 5. Anyone who thinks better technology is going to produce smarter students should spend a week arranging with another colleague to use a set of graphing calculators, checking them out and back in each period, switching calculators with good batteries for those with dead ones and later recharging the dead ones, and solving all the "my calculator is broken" problems that arise from strangely set ranges, modes, etc. that some malevolent elf contributed during the previous period. It leaves so much less time to teach math.

I do not know how many of the curricular problems I find in this integrated approach are unique to this text series. I do know that after some familiarity with probably fifty secondary mathematics texts in more than three decades, this is the only one which has ever made me ill. In light of that I've tried to be as objective as I can about what is and what is not there. I do have several very negative general impressions.

We jump around from topic to at best loosely related topic without enough connections to previous material or enough practice. Some students have a sense that we're dabbling in cute little curiosities that they have no need to master. In an early chapter, the authors suggest a timetable of two days on scientific notation, one on estimating area and length, two on angles (acute, right, obtuse, complementary, supplementary, vertical and in a triangle), one on combining like terms, one on solving equations, one on writing equations and undoing operations and two on roots. It still makes my head spin. When I studied foreign languages, words like "notte" and "Nacht" and "canale" and "Kanal" were easy to learn because they are so similar to "night" and "canal". "Ucèllo", "Vogel", "sécchia" and "Eimer" are much more difficult for me because they bear less resemblance to "bird" and "bucket". "Scientific notation", "vertical angles" and "roots" are probably that dissimilar and difficult for some students. I submit that the topics in the text we use are not integrated at all, just thrown into consecutive sections in an unrelated jumble.

Apparently it is politically correct to disdain much practice as "drill and kill", notwithstanding the numbers of times Michael Jordan and Larry Bird practiced free throws. But a student who does forty problems like $\frac{x^8}{x^2}$ is probably more likely to think it's important to do it accurately than one who has only done eight. One day one girl commented, almost in tears, that she finally thought she had the hang of something and now we were going on to something different. On many, many of the skills involved in algebra this approach does not offer enough problems for some students to feel they've successfully learned something. Of course it is possible to supplement the text with worksheets. I've run off 98 pages (on 49 sheets of paper) per student of worksheets for this text this year. So I've helped destroy some rain forest and helped cause floods and heatstroke deaths in the Midwest. There is no way I will ever feel good about this.

Perhaps it is expected that intermittent spiraling is a fine substitute for longer sets of problems, but I think the initial radius of the spiral is too small.

In place of simple examples and more practice problems, we get overwhelming clutter. Students are asked to research the Braille alphabet or gravity on other planets (for which they lack resources and time) or are asked why skateboarding is popular or told what the fuzz on a tennis ball accomplishes. If math has to be spiced up that way, I guess math is not as enjoyable as I always thought.

Because we leap from one topic to another, we often lack recourse to mathematical underpinnings. My students decided vertical angles are congruent because "they look like they are". Realizing I couldn't suggest the need for proof and establish within a few minutes that angles of a linear pair are supplementary and that supplements of the same angle are congruent, I shrugged and murmured something about perception not always being foolproof. When we did volumes and surface areas of pyramids, we just assumed an apothem (by name in a figure: we do not use the term) of a square is half as long as a side of the square so we could use it and a slant height to find the altitude by the Pythagorean Theorem. We never did areas or volumes of any other type of regular pyramid because we do not know vital material on congruent triangles, isosceles triangles, or regular polygons. When and where I received my degree, I understood a mathematical system to be a collection of elements, relations and operations whose properties are developed from definitions, postulates and theorems. This hodgepodge bears very little resemblance to that: I am propagating heresy.

If a student is absent or new to the school and trying to see how to do the problems on an assignment, this is a very difficult text to use. For three problems which ask for three solutions of each equation (in two variables), I looked back for help. I found that the preceding page shows pictures of plane tessellations with hexagons and triangles, a page before that has a line graph with intercepts and a sample of how to find intercepts, the page before that has a runner sometimes running and sometimes walking and an equation for her distance, and finally the page before that has a line graph and a paragraph that says what a solution of an equation is, but never says how to find them.

Two of the authors' other claims are that students can learn more mathematics than before and can have better retention of what they've learned. I couldn't use a standardized test this year: there are too many topics on it we've not covered. And retention is way down: I write tests which are like the problems we do in class and on assignments, but my failure rate has skyrocketed.

We've spent thousands of dollars on this sequence of texts because powerful people jumped on a bandwagon. I think they make learning and teaching mathematics much more difficult, and I have no idea what to do about it. Generally I just think about how soon I can retire........

Sincerely,
Diana Fogler
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